MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019

EEM2036 – ENGINEERING MATHEMATICS III

(ME/RE/TE)

15 OCTOBER 2018

9.00 A.M.– 11.00 A.M.

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This exam paper consists of 8 **printed pages** (including cover page and formula sheets) with **four questions** only.

2. For Question 1 and Question 2, answer 2 out of 3 parts only. Each part in Question 1 and 2 is 12 ½ marks. Question 3 and 4 are compulsory.

3. Please write all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

Part A

a) R is a triangle bounded by $y = x + 1$, $y = -x + 1$, and y=0. Write down the integral and state the Limits of Integration without solving it. [3 marks]

b) Evaluate $\iint_D e^{\frac{x}{y}} dA$, where D is the region between the curves $y = x$ and $x = y^3$ in the first quadrant. [9 ½ marks]

Part B

a) Let $u = \sqrt{x^2 + y^2}$ and $v = xy$. Compute the Jacobian $J(u, v)$. [3 marks]

b) Use $x = r\cos\theta$ and $y = r\sin\theta$ to evaluate

$I = \int_R e^{x^2+y^2} dxdy$

where R is the rectangle area with $1 \leq r \leq \sqrt{2}$ and $0 \leq \theta \leq \pi$. [9 ½ marks]

Part C

A wedge W in R^3 is given by:
Inside the cylinder $(x-1)^2 + y^2 = 1$, outside the cone $z^2 = x^2 + y^2$ and above the xy plane.

a) Convert $(x-1)^2 + y^2 = 1$ to polar coordinates. [2 marks]

b) Compute the volume of the wedge, i.e. $\iiint_W dV$.
Hint: $\int \cos^3\theta d\theta = \frac{1}{3}(2 + \cos^2\theta)\sin\theta$ [10 ½ marks]

Continued .....

Question 2

Part A

a) Determine whether the vector field

$$\vec{F} = (yz + e^x sinz)\hat{\imath} + (xz + y^2 - e^y)\hat{\jmath} + (xy + e^x cosz)\hat{k}$$

is conservative. If it is, find a function f such that $\vec{F} = \nabla f$. [9 ½ marks]

b) A particle moves by following the path xy^3 from (0,0,0) to (1,0,π/2) under the influence of the field $\vec{F}$. Find the work done of moving the particle.
[3 marks]

Part B

a) Compute divergence of $\vec{F}$, $div\ \vec{F}$, where $\vec{F} = (y + 2xe^y)\hat{\imath} + x^2e^y\hat{\jmath}$.
[4 marks]

b) By using Divergence Theorem, compute the outward flux of

$$\vec{F} = xy^2\hat{\imath} + y^3\hat{\jmath} + 4x^2z\hat{k}$$

through the surface of the (solid) cylinder W given by $x^2 + y^2 \leq 4, 0 \leq z \leq 5$.
(Hint: convert to polar coordinates). [8 ½ marks]

Part C

Evaluate the following vector field using Stokes' Theorem:

$$\vec{F} = yz\,\hat{\imath} - yz\,\hat{\jmath} + 2xy\,\hat{k}$$

The surface is enclosed by the close path $x^2 + y^2 = 1$ on the plane z =3.
[12 ½ marks]

Continued .....

Question 3

a) Suppose that a matrix A is given as follows: $A = \begin{bmatrix} 9 & -4 \\ 8 & -3 \end{bmatrix}$.

i) Show that the eigenvalues of A are 1 and 5 by formulating the characteristic equation.

ii) For one of the eigenvalue, find its eigenvector.

iii) Find the eigenvalues of A^{-1}.

[5+4+2 marks]

b) A system of differential equation is given as follows:

$$x_1' = 9x_1 - 4x_2$$
$$x_2' = 8x_1 - 3x_2$$

where x_1 and x_2 are functions of t. Given that the eigenvalues of $A = \begin{bmatrix} 9 & -4 \\ 8 & -3 \end{bmatrix}$ are 1 and 5 with the corresponding eigenvectors $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, respectively.

i) Write a matrix P that diagonalizes A.

ii) Write a diagonal matrix D such that $P^{-1}AP = D$.

iii) Hence, find the general solution to the system.

[2+2+10 marks]

Continued .....

Question 4

a) Find the cubic polynomial $P_3(x)$ for the following data using Lagrange interpolating polynomial.

x	-2	0	2	4
$f(x)$	15	9	-5	21

[13 marks]

b) Given the following initial value differential equation:

$$y' - y = x, \quad \text{for } 0 \leq x \leq 1 \quad \text{with } y(0) = 0 \quad \text{and } h = 0.2$$

Compute y_1, y_2 and y_3 (5 decimal places) by using Runge-Kutta second order method.

[12 marks]

Continued ....

APPENDIX

TABLE OF FORMULAS

1. The nth Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k](x - x_0) \Lambda\ (x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

with the error term

$$\left| \frac{h^2}{6} f^{(3)}(c_x) \right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(a) + f(b)]$$

for some ξ in (a, b) and $h = b - a$, with the error term is $\left| \frac{h^3 f''(\xi)}{12} \right|$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{j=1}^{n-1} f(x_j) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term is $\left| \frac{(b-a)h^2 f''(\xi)}{12} \right|$.

10. Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{2}$, with the error term $\left| \frac{h^5}{90} f^{(4)}(\xi) \right|$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j}) + 4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1}) + f(b) \right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term $\left| \frac{(b-a)h^4}{180} f^{(4)}(\xi) \right|$.

Continued ...

12. Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n = 0,1,2,...$$

13. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2}Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

14. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

15. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1),$$
$$k_3 = hf(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

End of Paper